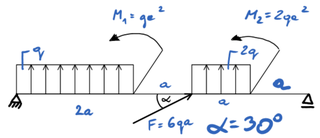
KOLOKWIUM

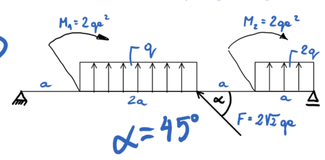
środa, 20 kwietnia 2016
07:58

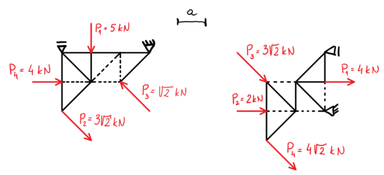
$\alpha = 30°$

A

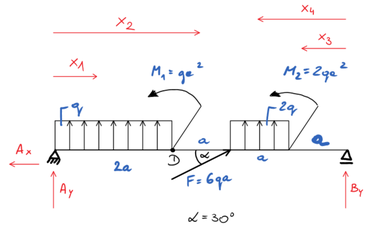


B





Wyznacz SIŁY WEWNĘTRZ pętach oznaczonych linią przerywaną - - - - -



$\alpha = 30°$

① $\Sigma F_{ix}: \; -A_x + F\cos\alpha = 0$

② $\Sigma F_{iy}: \; A_y + 2qa + F\sin\alpha + 2qa + B_y = 0$

③ $\Sigma M_i^A: \; -2qa\cdot a - M_1 - F\sin\alpha\cdot 3a - 2qa\cdot 3{,}5a - M_2 - B_y\cdot 5a = 0$

Z równania ③:

$-2qa^2 - qa^2 - 9qa^2 - 7qa^2 - 2qa^2 = 5aB_y$

$-21qa^2 = 5aB_y$

$B_y = -\frac{21}{5}qa = -4\frac{1}{5}qa$

Z równania ②:

$A_y + 2qa + 3qa + 2qa - 4\frac{1}{5}qa = 0$

$A = -2\frac{4}{5}qa = -\frac{14}{5}qa$

SPRAWDZENIE:

$\Sigma M_i^D: \; A_y\cdot 2a + 2qa\cdot a - M_1 - F\sin\alpha\cdot a - 2qa\cdot 1{,}5a - M_2 - B_y\cdot 3a = 0$

$-\frac{28}{5}qa^2 + 2qa^2 - qa^2 - 3qa^2 - 3qa^2 - 2qa^2 + \frac{63}{5}qa^2 = 0$

$0 = 0$ ok!

$A_y = -\frac{14}{5}qa \qquad B_y = -\frac{21}{5}qa$

OD LEWEJ

$Mg^{I} = A_y x_1 + \frac{1}{2}q x_1^2$

$T^{I} = A_y + q x_1$

$Mg^{I}(x_1 = 0) = 0$

$Mg^{I}(x_1 = 2a) = -\frac{28}{5}qa^2 + 2qa^2 = -\frac{18}{5}qa^2$

$T^{I}(x_1 = 0) = -\frac{14}{5}qa$

$T^{I}(x_1 = 2a) = -\frac{14}{5}qa + 2qa = -\frac{4}{5}qa$

$Mg^{II} = A_y x_2 + 2qa(x_2 - a) - M_1$

$T^{II} = A_y + 2qa$

$Mg^{II}(x_2 = 2a) = -\frac{28}{5}qa^2 + 2qa^2 - qa^2 = -\frac{23}{5}qa^2$

$Mg^{II}(x_2 = 3a) = -\frac{42}{5}qa^2 + 4qa^2 - qa^2 = -\frac{27}{5}qa^2$

$T^{II} = -\frac{4}{5}qa$

OD PRAWEJ:

$Mg^{III} = B_y x_3$

$T^{III} = -B_y$

$Mg^{III}(x_3 = 0) = 0$

$Mg^{III}(x_3 = a) = -\frac{21}{5}qa^2$

$T^{III} = \frac{21}{5}qa$

$Mg^{IV} = B_y x_4 + M_2 + q(x_4 - a)^2$

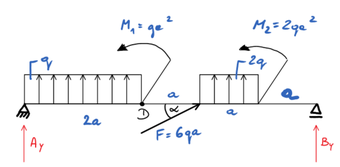
$T^{IV} = -B_y - 2q(x_4 - a)$

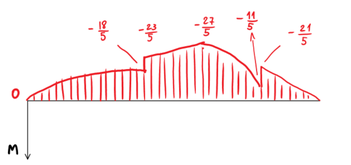
$Mg^{IV}(x_4 = a) = -\frac{21}{5}qa^2 + 2qa^2 = -\frac{11}{5}qa^2$

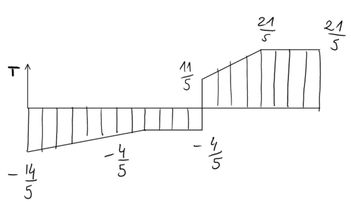
$Mg^{IV}(x_4 = 2a) = -\frac{42}{5}qa^2 + 2qa^2 + qa^2 = -\frac{27}{5}qa^2$

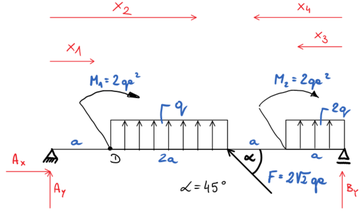
$T^{IV}(x_4 = a) = \frac{21}{5}qa$

$T^{IV}(x_4 = 2a) = \frac{21}{5}qa - 2qa = \frac{11}{5}qa$









① $\Sigma F_{ix}: \; A_x - F\cos\alpha = 0$

② $\Sigma F_{iy}: \; A_y + 2qa + F\sin\alpha + 2qa + B_y = 0$

③ $\Sigma M_i^A: \; M_1 - 2qa\cdot 2a - F\sin\alpha\cdot 3a + M_2 - 2qa\cdot 4{,}5a - B_y\cdot 5a = 0$

Z równania ③:

$2qa^2 - 4qa^2 - 6qa^2 + 2qa^2 - 9qa^2 = 5aB_y$

$B_y = -3qa$

Z równania ②:

$A_y + 2qa + 2qa + 2qa + B_y = 0$

$A_y = -3qa$

SPRAWDZENIE:

$\Sigma M_i^D: \; A_y\cdot a + M_1 - 2qa\cdot a - F\sin\alpha\cdot 2a + M_2 - 2qa\cdot 3{,}5a - B_y\cdot 4a = 0$

$-3qa^2 + 2qa^2 - 2qa^2 - 4qa^2 + 2qa^2 - 7qa^2 + 12qa^2 = 0$

$0 = 0$ ok!

$A_y = -3qa \qquad B_y = -3qa$

OD LEWEJ:

$Mg^{I} = A_y x_1$

$T^{I} = A_y$

$Mg^{I}(x_1 = 0) = 0$

$Mg^{I}(x_1 = a) = -3qa^2$

$T^{I} = -3qa$

$Mg^{II} = A_y x_2 + M_1 + \frac{1}{2}q(x_2 - a)^2$

$T^{II} = A_y + q(x_2 - a)$

$Mg^{II}(x_2 = a) = -3qa^2 + 2qa^2 = -qa^2$

$Mg^{II}(x_2 = 3a) = -9qa + 2qa^2 + 2qa^2 = -5qa^2$

$T^{II}(x_2 = a) = -3qa$

$T^{II}(x_2 = 3a) = -3qa + 2qa = -qa$

OD PRAWEJ:

$Mg^{III} = B_y x_3 + q x_3^2$

$T^{III} = -B_y - 2qx_3$

$Mg^{III}(x_3 = 0) = 0$

$Mg^{III}(x_3 = a) = -3qa^2 + qa^2 = -2qa^2$

$T^{III}(x_3 = 0) = -B_y = 3qa$

$T^{III}(x_3 = a) = 3qa - 2qa = qa$

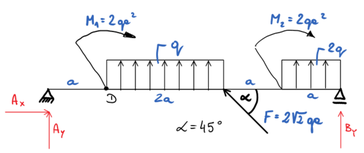
$Mg^{IV} = B_y x_4 + 2qa\left(x_4 - \frac{a}{2}\right) - M_2$

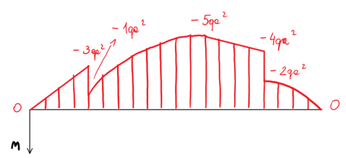
$T^{IV} = -B_y - 2qa$

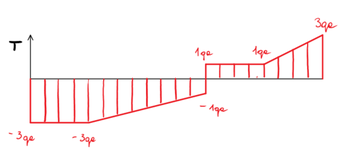
$Mg^{IV}(x_4 = a) = -3qa^2 + qa^2 - 2qa^2 = -4qa^2$

$Mg^{IV}(x_4 = 2a) = -6qa^2 + 3qa^2 - 2qa^2 = -5qa^2$

$T^{IV} = 3qa - 2qa = qa$







$P_1 + P_{2x} - B_x - P_{3x} = 0$

$-A_y - P_{2y} - P_1 + P_{3y} - B_y = 0$

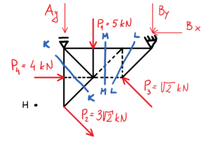
$\Sigma M^A: \; -P_1 a - P_2 a\sqrt{2} - P_3 a\frac{\sqrt{2}}{2} + B_y 2a = 0$

$-P_1 + P_1 - P_2\sqrt{2} - P_3\frac{\sqrt{2}}{2} = -3B_y$

$-4 + 5 - 6 - 1 = -3B_y$

$-6 = -3B_y$

$B_y = 2$



① $\Sigma F_{ix}: \; P_4 + P_2\sin 45° - P_3\sin 45° - B_x = 0$

② $\Sigma F_{iy}: \; -A_y - B_y - P_1 - P_2\sin 45° + P_3\sin 45° = 0$

③ $\Sigma M_i^A: \; -P_4 a - P_2 a\sqrt{2} - P_3\frac{a\sqrt{2}}{2} + P_1 a + B_y\cdot 3a = 0$

Z równania ①

$B_x = P_4 + P_2\sin 45° - P_3\sin 45°$

$B_x = 4 + 3 - 1$

$B_x = 6\ kN$

Z równania ③

$3B_y = P_4 + P_2\sqrt{2} + P_3\frac{\sqrt{2}}{2} - P_1$

$3B_y = 4 + 6 + 1 - 5$

$B_y = 2\ kN$

Z równania ②

$A_y = -B_y - P_1 - P_2\sin 45° + P_3\sin 45°$

$A_y = -2 - 5 - 3 + 1$

$A_y = -9\ kN$

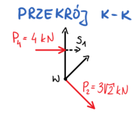
SPRAWDZENIE:

$\Sigma M_i^M: \; A_y a + P_1 2a + B_y 4a - B_x 2a + P_4 a + P_2\frac{a\sqrt{2}}{2} - P_3 2a\sqrt{2} = 0$

$-9 + 10 + 8 - 12 + 4 + 3 - 4 = 0$

$-21 + 21 = 0$ ok!

PRZEKRÓJ K-K

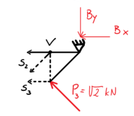


$\Sigma M_i^N: \; P_4 a + S_1 a = 0$

$S_1 = -P_4$

$S_1 = -4kN$

PRZEKRÓJ M-M



$\Sigma M_i^V: \; S_3 a + P_3\frac{a\sqrt{2}}{2} + B_y a = 0$

$S_3 = -B_y - P_3\frac{\sqrt{2}}{2}$

$S_3 = -2 - 1$

$S_3 = -3\ kN$

$\Sigma F_{iy}: \; -S_{2y} + P_{3y} - B_y = 0$

$S_{2y} = 1 - 2$

$S_{2y} = -1\ kN$

$S_2 = -\sqrt{2}\ kN$

PRZEKRÓJ L-L

$\Sigma F_{iy}: \; S_4 + P_{3y} - B_y = 0$

$S_4 = B_y - P_{3y}$

$S_4 = 2 - 1$

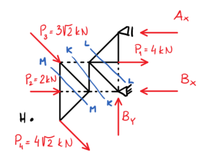
$S_4 = 1\ kN$

$S_1 = 4kN$ ściskany

$S_2 = -\sqrt{2}kN$ ściskany

$S_3 = 3kN$ ściskany

$S_4 = 1\ kN$ rozciągany



① $\Sigma F_{ix}: \; -A_x - B_x + P_1 + P_3\sin 45° + P_2 + P_4\sin 45° = 0$

② $\Sigma F_{iy}: \; B_y - P_3\sin 45° - P_4\sin 45° = 0$

③ $\Sigma M_i^B: \; P_1 a - P_3\frac{a\sqrt{2}}{2} - P_4\frac{3a\sqrt{2}}{2} - A_x 2a = 0$

Z równania ②:

$B_y = P_3\sin 45° + P_4\sin 45°$

$B_y = 3 + 4$

$B_y = 7\ kN$

Z równania ③:

$2A_x = P_1 - P_3\frac{\sqrt{2}}{2} - P_4\frac{3\sqrt{2}}{2}$

$2A_x = 4 - 3 - 12$

$A_x = -5{,}5\ kN$

Z równania ①:

$B_x = -A_x + P_1 + P_3\frac{\sqrt{2}}{2} + P_2 + P_4\frac{\sqrt{2}}{2}$

$B_x = 5{,}5 + 4 + 3 + 2 + 4$

$B_x = 18{,}5\ kN$

SPRAWDZENIE:

$\Sigma M_i^H: \; P_4\frac{a\sqrt{2}}{2} + P_2 a + P_3\frac{3a\sqrt{2}}{2} - 3aB_y - aB_x + 2aP_1 - A_x 3a = 0$

$4\sqrt{2}\cdot\frac{\sqrt{2}}{2} + 2 + 3\sqrt{2}\cdot\frac{3\sqrt{2}}{2} - 3\cdot 7 - 18{,}5 + 8 + 16{,}5 = 0$

$4 + 2 + 9 - 21 - 18{,}5 + 8 + 16{,}5 = 0$

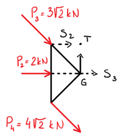
$39{,}5 - 39{,}5 = 0$ ok!

PRZEKRÓJ M-M

$\Sigma M_i^R: \; P_2 a + S_1 a = 0$

$S_1 = -P_2$

$S_1 = -2kN$



$\Sigma M_i^G: \; S_2 a - P_4 a\sqrt{2} = 0$

$S_2 = P_4\sqrt{2}$

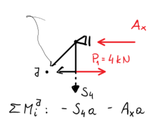
$S_2 = 8\ kN$

$\Sigma M_i^T: \; -S_3 a - P_2 a - P_4\cdot 1{,}5\sqrt{2}a - P_3\cdot 0{,}5\sqrt{2}a = 0$

$S_3 = -(P_2 + 1{,}5\sqrt{2}P_4 + 0{,}5\sqrt{2}P_3)$

$S_3 = -(2 + 12 + 3)\ kN$

$S_3 = -17\ kN$



$\Sigma M_i^J: \; -S_4 a - A_x a = 0$

$S_4 = -A_x$

$S_4 = 5{,}5\ kN$