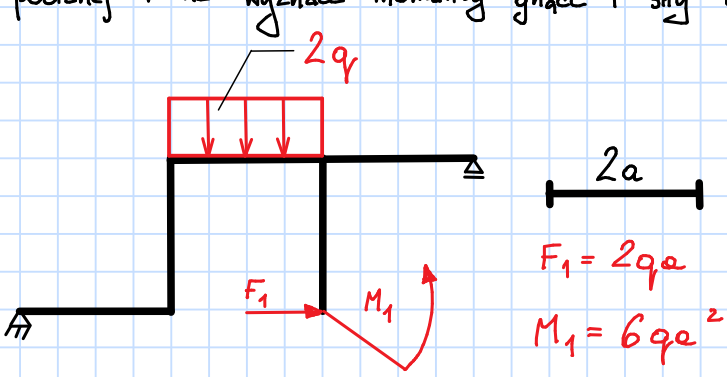
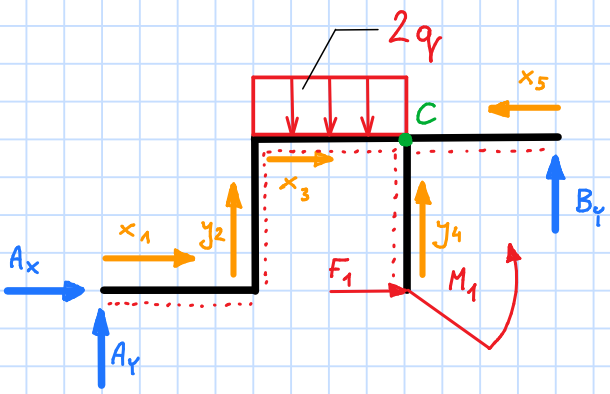
GRUPA A

1. W podanej ramie wyznacz momenty gnące i siły tnące. Przygotuj szkic tych wielkości.



$F_1 = 2qa$

$M_1 = 6qa^2$



Wyznaczenie reakcji:

$\Sigma F_x: \quad A_x + F_1 = 0$

$\Sigma F_y: \quad A_y + B_y - 2q \cdot 2a = 0$

$\Sigma M^A: \quad 2q \cdot 2a \cdot 3a - M_1 - 6a \cdot B_y = 0$

$A_x = -F_1 = -2qa$

$12qa^2 - 6qa^2 = 6aB_y$

$B_y = qa$

$A_y = 4qa - qa$

$A_y = 3qa$

SPRAWDZENIE:

$A_y \cdot 4a - A_x \cdot 2a - 2q \cdot 2a \cdot a - F_1 \cdot 2a - M_1 - B_y \cdot 2a = 0$

$12qa^2 + 4qa^2 - 4qa^2 - 4qa^2 - 6qa^2 - 2qa^2 = 0$ ok!

Przedział I

$Mg^{I} = A_y \cdot x_1 \qquad Mg^{I}(x_1 = 2a) = 6qa^2$

$T^{I} = A_y = 3qa$

Przedział II

$Mg^{II} = Mg^{I}(x_1 = 2a) - A_x \cdot y_2$

$Mg^{II}(y_2 = 0) = 6qa^2$

$Mg^{II}(y_2 = 2a) = 6qa^2 + 4qa^2 = 10qa^2$

$T^{II} = -A_x = 2qa$

Przedział III

$Mg^{III} = Mg^{II}(y_2 = 2a) + A_y x_3 - 2q x_3 \cdot \frac{x_3}{2}$

$Mg^{III}(x_3 = 2a) = 10qa^2 + 6qa^2 - 4qa^2 = 12qa^2$

$T^{III} = A_y - 2q x_3$

$T^{III}(x_3 = 0) = A_y = 3qa$

$T^{III}(x_3 = 2a) = A_y - 4qa = -qa$

Przedział IV

$Mg^{IV} = M_1 + F_1 y_4$

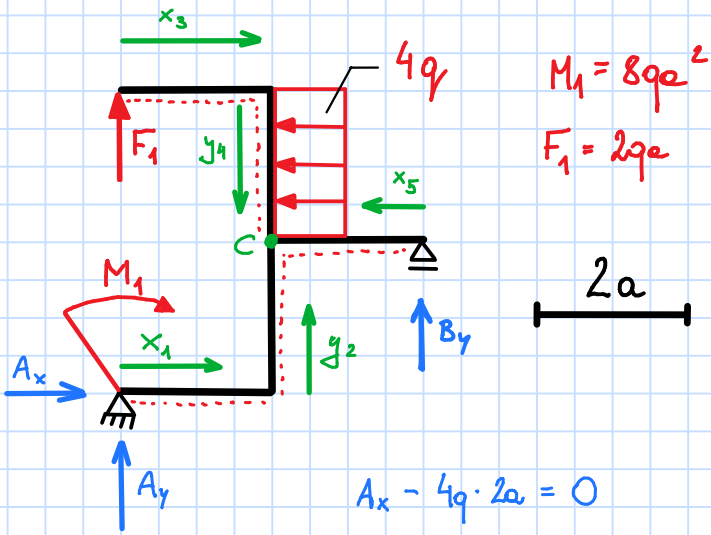
$Mg^{IV}(y_4 = 0) = 6qa^2 \qquad Mg^{IV}(y_4 = 2a) = 10qa^2$

$T^{IV} = -F_1 = -2qa$

Przedział V

$Mg^{V} = B_y x_5 \qquad Mg^{V}(x_5 = 2a) = 2qa^2$

$T^{V} = -B_y = -qa$



$M_1 = 8qa^2$

$F_1 = 2qa$

$A_x - 4q \cdot 2a = 0 \qquad A_x = 8qa$

$A_y + B_y + F_1 = 0$

$\Sigma M^A: \quad M_1 - 4q \cdot 2a \cdot 3a = 4a B_y$

$8qa^2 - 24qa^2 = 4a B_y$

$-16qa^2 = 4a B_y$

$B_y = -4qa$

$A_y = -B_y - F_1 = 4qa - 2qa = 2qa$

SPRAWDZENIE wg punktu C

$-B_y \cdot 2a - 4q \cdot 2a \cdot a + F_1 \cdot 2a + A_y \cdot 2a - A_x \cdot 2a + M_1 = 0$

$+8qa^2 - 8qa^2 + 4qa^2 + 4qa^2 - 16qa^2 + 8qa^2 = 0$ ok!

PRZEDZIAŁ I

$Mg^I = A_y \cdot x_1 + M_1$

$Mg^I(x_1 = 0) = M_1 = 8qa^2$

$Mg^I(x_1 = 2a) = 2qa \cdot 2a + 8qa^2 = 12qa^2$

$T^I = A_y = 2qa$

PRZEDZIAŁ II

$My^{II} = Mg^I(x_1 = 2a) - A_x \cdot y_2$

$Mg^{II}(y_2 = 0) = 12qa^2$

$Mg^{II}(y_2 = 2a) = 12qa^2 - 8qa \cdot 2a = -4qa^2$

$T^{II} = -A_x = -8qa$

PRZEDZIAŁ III

$Mg^{III} = F_1 \cdot x_3$

$Mg^{III}(x_3 = 2a) = 2qe \cdot 2a = 4qe^2$

$T^{III} = F_1 = 2qe$

PRZEDZIAŁ IV

$Mg^{IV} = Mg^{III}(x_3 = 2a) - 4q\frac{y_4}{2}^2 = 4qe^2 - 2qy_4^2$

$Mg^{IV}(y_4 = 2a) = 4qe^2 - 8qe^2 = -4qe^2$

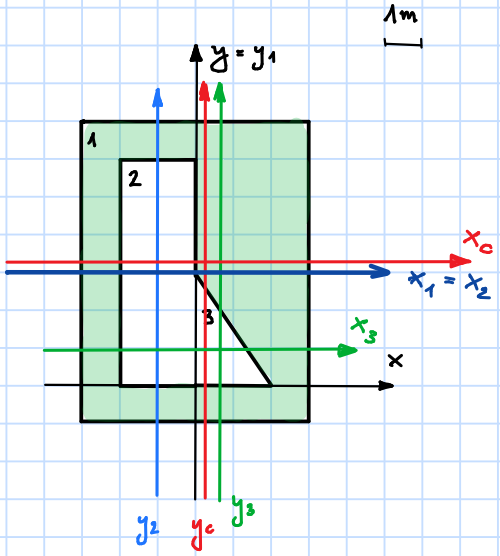
$T^{IV} = -4qy_4$ $\quad T^{IV}(y_4 = 0) = 0$ $\quad T^{IV}(y_4 = 2a) = -8qe$

PRZEDZIAŁ V

$Mg^{V} = B_y \cdot x_5$ $\quad Mg^{V}(x_5 = 2a) = B_y \cdot 2a = -4qe \cdot 2a = -8qe^2$

$T^{V} = -B_y = 4qe$

2. Dla przedstawionej figury płaskiej wyznacz położenie środka ciężkości oraz momenty bezwładności względem osi X i Y przechodzących przez ten punkt.



$x_1 = 0$ $\quad y_1 = 3$ $\quad A_1 = 48$

$x_2 = -1$ $\quad y_2 = 3$ $\quad A_2 = 12$

$x_3 = \frac{2}{3}$ $\quad y_3 = 1$ $\quad A_3 = 3$

$$x_c = \frac{0 \cdot 48 + (-1) \cdot (-12) + \frac{2}{3} \cdot (-3)}{48 - 12 - 3} = \frac{12 - 2}{33} = \frac{10}{33} = 0,(30)$$

$$y_c = \frac{3 \cdot 48 + 3 \cdot (-12) + 1 \cdot (-3)}{48 - 12 - 3} = 3,(18)$$

$$J_x^1 = \frac{6 \cdot 8^3}{12} + 48 \cdot (3 - 3,18)^2 \approx 257,6$$

$$J_x^2 = \frac{2 \cdot 6^3}{12} + 12 \cdot (3 - 3,18)^2 \approx 36,4$$

$$J_x^3 = \frac{2 \cdot 3^3}{36} + 3 \cdot (1 - 3,18)^2 \approx 15,8$$

$$J_{x_c} = J_x^1 - J_x^2 - J_x^3 = 205,4 \ m^4$$

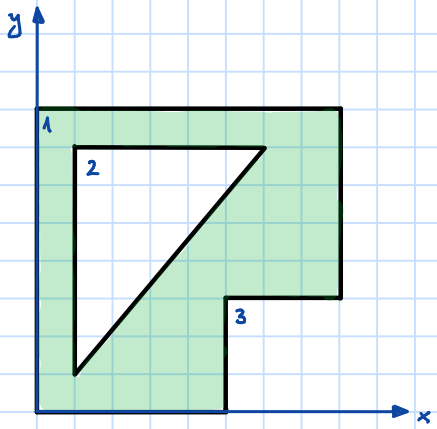
$$J_y^1 = \frac{8 \cdot 6^3}{12} + 48 \cdot (0 - 0,3)^2 \approx 148,3$$

$$J_y^2 = \frac{6 \cdot 2^3}{12} + 12 \cdot (-1 - 0,3)^2 \approx 24,3$$

$$J_y^3 = \frac{3 \cdot 2^3}{36} + 3 \left(\frac{2}{3} - 0,3\right)^2 \approx 1,1$$

$$J_{y_c} = J_y^1 - J_y^2 - J_y^3 = 122,9$$

2. Dla przedstawionej figury płaskiej wyznacz położenie środka ciężkości oraz momenty bezwładności względem osi X i Y przechodzących przez ten punkt.



$x_1 = 4 \quad y_1 = 4 \quad A_1 = 64$

$x_2 = 1 + \frac{5}{3} = \frac{8}{3} \quad y_2 = 5 \quad A_2 = 15$

$x_3 = 6{,}5 \quad y_3 = 1{,}5 \quad A_3 = 9$

$$x_c = \frac{4 \cdot 64 - \frac{8}{3} \cdot 15 - 6{,}5 \cdot 9}{64 - 15 - 9} \simeq 3{,}94$$

$$y_c = \frac{4 \cdot 64 - 5 \cdot 15 - 1{,}5 \cdot 9}{64 - 15 - 9} \simeq 4{,}19$$

$$J_x^1 = \frac{8 \cdot 8^3}{12} + 64 \cdot (4 - 4{,}19)^2 \simeq 343{,}6 \ m^4$$

$$J_x^2 = \frac{5 \cdot 6^3}{36} + 15 \cdot (5 - 4{,}19)^2 \simeq 39{,}8 \ m^4$$

$$J_x^3 = \frac{3 \cdot 3^3}{12} + 9 \cdot (1{,}5 - 4{,}19)^2 \simeq 71{,}9 \ m^4$$

$$J_{x_c} = J_x^1 - J_x^2 - J_x^3 = 231{,}9 \ m^4$$

$$J_y^1 = \frac{8 \cdot 8^3}{12} + 64 \cdot (4 - 3{,}94)^2 \simeq 341{,}6 \ m^4$$

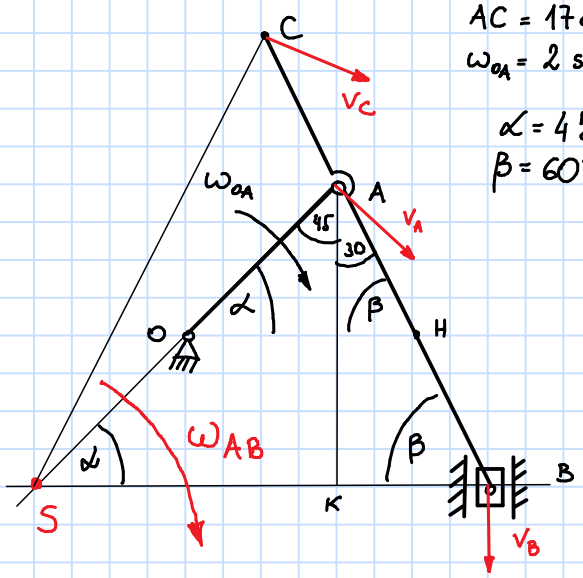
$$J_y^2 = \frac{6 \cdot 5^3}{36} + 15 \cdot \left(\frac{8}{3} - 3{,}94\right)^2 \simeq 45{,}2 \ m^4$$

$$J_y^3 = \frac{3 \cdot 3^3}{12} + 9 \cdot (6{,}5 - 3{,}94)^2 \simeq 65{,}7 \ m^4$$

$$J_{y_c} = J_y^1 - J_y^2 - J_y^3 = 230{,}7 \ m^4$$

$OA = 20\ cm$
$AB = 35\ cm$
$AC = 17\ cm$
$\omega_{OA} = 2\ s^{-1}$

$\alpha = 45°$
$\beta = 60°$



$$v_A = \omega_{OA} \cdot OA = \omega_{AB} \cdot AS = 40\ cm/s$$

$$\omega_{AB} = \frac{\omega_{OA} \cdot OA}{AS} = \frac{2 \cdot 20}{42,9} \simeq 0,93$$

$$\frac{OA}{AH} = \frac{AS}{AB} \qquad \frac{AK}{AB} = \sin\beta$$

$$AK = AB \sin\beta$$

$$AK = 35 \cdot \frac{\sqrt{3}}{2}$$

$$\frac{AK}{AS} = \sin\alpha$$

$$AS = \frac{AK}{\sin\alpha} = \frac{35\sqrt{3}}{2} \cdot \frac{2}{\sqrt{2}} = \frac{70\sqrt{3}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$AS = \frac{70\sqrt{6}}{4} = 17,5\sqrt{6} \simeq 42,9\ cm$$

$$BS = \sqrt{AS^2 + AB^2 - 2 \cdot AS \cdot AB \cdot \cos 75°}$$

$$BS = \sqrt{(17,5\sqrt{6})^2 + 35^2 - 2 \cdot 17,5\sqrt{6} \cdot 35 \cdot \cos 75°}$$

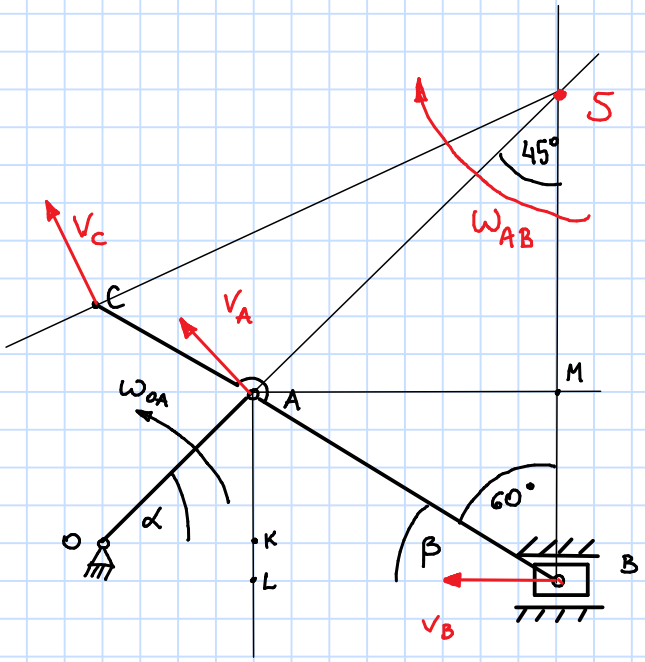
$$BS = 47,8\ cm$$

$$CS = \sqrt{BS^2 + BC^2 - 2 \cdot BS \cdot BC \cdot \cos\beta}$$

$$CS = 50\ cm$$

$$v_C = CS \cdot \omega_{AB} = 50 \cdot 0,93 \simeq 46,5\ cm/s$$

$$v_B = BS \cdot \omega_{AB} = 47,8 \cdot 0,93 \simeq 44,5\ cm/s$$



$OA = 10\,cm$
$AB = 21\,cm$
$AC = 8\,cm$
$\omega_{OA} = \frac{1}{2}\,s^{-1}$

$\alpha = 45°$
$\beta = 30°$

$$v_A = OA \cdot \omega_{OA} = AS \cdot \omega_{AB}$$

$$\omega_{AB} = \frac{\omega_{OA} \cdot OA}{AS} \simeq 0{,}2\,s^{-1}$$

$$\frac{AM}{AB} = \sin 60°$$

$$AM = AB \sin 60° = 21 \cdot \frac{\sqrt{3}}{2}\,cm = 10{,}5\sqrt{3}\,cm$$

$$\frac{AM}{AS} = \sin 45°$$

$$AS = \frac{AM}{\sin 45°} = 10{,}5\sqrt{3} \cdot \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = 10{,}5\sqrt{6} \simeq 25{,}7\,cm$$

$$BS = SM + BM$$

$$BS = AS \sin 45° + AB \sin 30°$$

$$= 10{,}5\sqrt{6} \cdot \frac{\sqrt{2}}{2} + 21 \cdot 0{,}5$$

$$= 10{,}5\sqrt{3} + 10{,}5 = 10{,}5(\sqrt{3}+1)\,cm \simeq 28{,}7\,cm$$

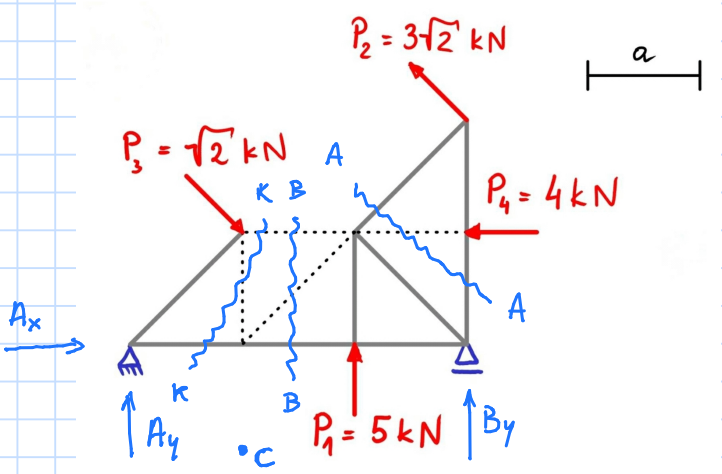
$$CS = \sqrt{CB^2 + BS^2 - 2 \cdot CB \cdot BS \cdot \cos 60°}$$

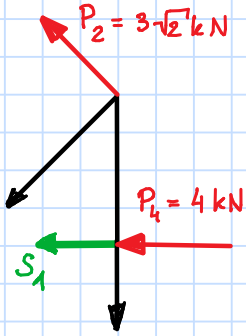
$$CS = \sqrt{29^2 + 28{,}7^2 - 2 \cdot 29 \cdot 28{,}7 \cdot \frac{\sqrt{3}}{2}}$$

$$CS \simeq 28{,}85\,cm$$

$$v_B = \omega_{AB} \cdot BS \simeq 5{,}7\,cm/s$$

$$v_C = \omega_{AB} \cdot CS \simeq 5{,}8\,cm/s$$





$\Sigma F_x = 0$: $\quad A_x + P_{3x} - P_{2x} - P_4 = 0$

$\Sigma F_y = 0$: $\quad A_y + B_y + P_1 - P_{3y} + P_{2y} = 0$

$\Sigma M^A = 0$: $\quad P_3 \cdot a\sqrt{2} - 2aP_1 - P_2 \cdot 2{,}5a\sqrt{2} - P_4 \cdot a = 3aB_y$

z 3 równania:

$\sqrt{2} \cdot a\sqrt{2} - 2a \cdot 5 - 3\sqrt{2} \cdot 2{,}5a\sqrt{2} - 4a = 3aB_y \;/ : a$

$2 - 10 - 15 - 4 = 3B_y$

$-27 = 3B_y$

$B_y = -9$

z 1 równania:

$A_x = P_4 + P_{2x} - P_{3x}$

$A_x = 4 + 3 - 1$

$A_x = 6$

z 2 równania:

$A_y = -B_y - P_1 + P_{3y} - P_{2y}$

$A_y = 9 - 5 + 1 - 3$

$A_y = 2$

SPRAWDZENIE: (względem punktu C)

$A_x \cdot a + A_y \cdot a + P_3 a\sqrt{2} - P_1 a - 2aB_y - 2aP_4 - P_3 \cdot 2{,}5a\sqrt{2} = 0$

$6a + 2a + 2a - 5a + 18a - 8a - 15a = 0$ ok!