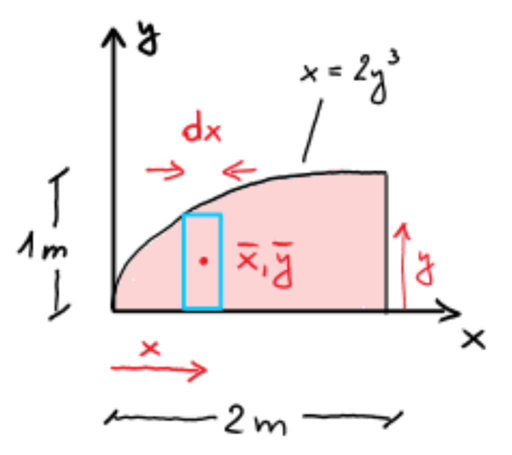
Dla przedstawionej figury znajdź pole powierzchni, środek ciężkości oraz wyznacz momenty bezwładności względem osi x i y przyjętego układu.



Powierzchnia elementarna: $dA = y\,dx$

Środek ciężkości powierzchni elementarnej: $\bar{x} = x$, $\bar{y} = \frac{1}{2}y$

Pole powierzchni figury

$A = \int_A dA$ $\qquad x = 2y^3 \Rightarrow y = \left(\frac{x}{2}\right)^{\frac{1}{3}}$

$$A = \int_0^2 y\,dx = \int_0^2 \left(\frac{x}{2}\right)^{\frac{1}{3}} dx = \sqrt[3]{\tfrac{1}{2}} \int_0^2 x^{\frac{1}{3}} dx = \sqrt[3]{\tfrac{1}{2}} \cdot \frac{3}{4} \cdot x^{\frac{4}{3}} \Big|_0^2$$

$A = 1{,}5\ m^2$

$$\bar{x} = \frac{\int_A \bar{x}\,dA}{\int_A dA} = \frac{\int_0^2 x y\,dx}{\int_0^2 y\,dx} \quad \frac{\int_0^2 x \cdot \left(\frac{1}{2}x\right)^{\frac{1}{3}} dx}{1{,}5} = \frac{\sqrt[3]{\frac{1}{2}} \int_0^2 x^{\frac{4}{3}} dx}{1{,}5}$$

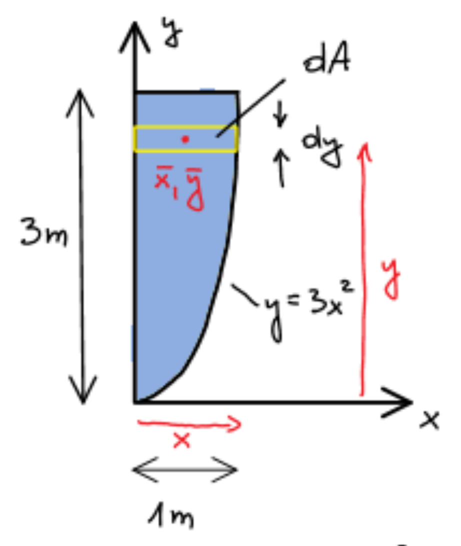
$$\bar{x} = \frac{\sqrt[3]{\frac{1}{2}} \cdot \frac{3}{7} \cdot x^{\frac{7}{3}} \Big|_0^2}{1{,}5} = 1{,}143\,m$$

$$\bar{y} = \frac{\int_A \bar{y}\,dA}{\int_A dA} = \frac{\int_0^2 \frac{1}{2}y \cdot y\,dx}{\int_0^2 y\,dx} = \frac{\frac{1}{2}\int_0^2 y^2\,dx}{1{,}5}$$

$$\bar{y} = \frac{\frac{1}{2}\int_0^2 \left(\frac{x}{2}\right)^{\frac{2}{3}} dx}{1{,}5} = \frac{\frac{1}{2} \cdot \left(\frac{1}{2}\right)^{\frac{2}{3}} \cdot \int_0^2 x^{\frac{2}{3}} dx}{1{,}5} = \frac{\frac{1}{2} \cdot \frac{1}{2}^{\frac{2}{3}} \cdot \frac{3}{5} x^{\frac{5}{3}} \Big|_0^2}{1{,}5} = 0{,}4\,m$$

$$I_x = \int_A y^2 dA = \int_0^1 y^2 (2-x)\,dy = \int_0^1 y^2 (2 - 2y^3)\,dy = \int_0^1 (2y^2 - 2y^5)\,dy = 2\left(\tfrac{1}{3}y^3 - \tfrac{1}{6}y^6\right)\Big|_0^1 = \tfrac{1}{3}\,m^4$$

$$I_y = \int_A x^2 dA = \int x^2 y\,dx = \int_0^2 x^2 \left(\frac{x}{2}\right)^{\frac{1}{3}} dx = \sqrt[3]{\tfrac{1}{2}} \int_0^2 x^2 \cdot x^{\frac{1}{3}} dx = \sqrt[3]{\tfrac{1}{2}} \int_0^2 x^{\frac{7}{3}} dx = \sqrt[3]{\tfrac{1}{2}}\, \tfrac{3}{10} x^{\frac{10}{3}} \Big|_0^2 = 2{,}4\,m^4$$



Powierzchnia elementarna: $dA = x\,dy$

Środek ciężkości powierzchni elementarnej: $\bar{x} = \frac{1}{2}x$, $\bar{y} = y$

Pole powierzchni figury

$A = \int_A dA$ $\qquad y = 3x^2 \qquad x = \left(\frac{y}{3}\right)^{\frac{1}{2}}$

$$A = \int_0^3 x\,dy = \sqrt{\tfrac{1}{3}} \int_0^3 y^{\frac{1}{2}} dy = \sqrt{\tfrac{1}{3}} \cdot \tfrac{2}{3} \cdot y^{\frac{3}{2}} \Big|_0^3 = 2\,m^2$$

$$\bar{x} = \frac{\int_A \bar{x}\,dA}{\int_A dA} = \frac{\int_0^3 \frac{1}{2}x \cdot x\,dy}{2} = \frac{\frac{1}{2}\int_0^3 x^2 dy}{2} = \frac{\frac{1}{2}\int_0^3 \frac{y}{3} dy}{2} = \frac{\frac{1}{6} \cdot \frac{1}{2} y^2 \Big|_0^3}{2} = 0{,}375\,m$$

$$\bar{y} = \frac{\int_A \bar{y}\,dA}{\int_A dA} = \frac{\int_0^3 y x\,dy}{2} = \frac{\int_0^3 3x^2 x\,dy}{2} = \frac{3\int_0^3 x^3 dy}{2} = \frac{3\int_0^3 \left(\frac{y}{3}\right)^{\frac{3}{2}} dy}{2}$$

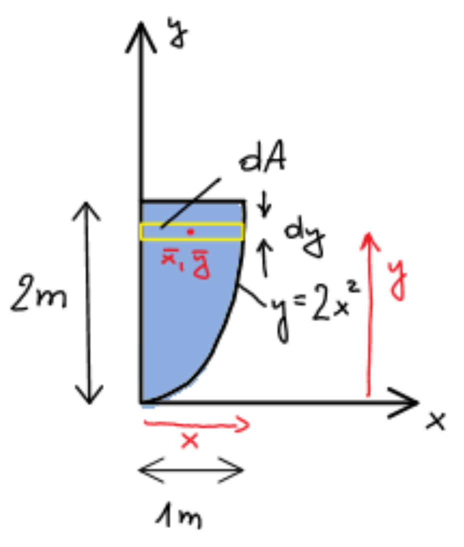
$$= \frac{3 \cdot \left(\frac{1}{3}\right)^{\frac{3}{2}} \int_0^3 y^{\frac{3}{2}} dy}{2} = \frac{3 \cdot \left(\frac{1}{3}\right)^{\frac{3}{2}} \cdot \frac{2}{5} \cdot y^{\frac{5}{2}} \Big|_0^3}{2} = 1{,}8\,m$$

$$I_x = \int_A y^2 dA = \int_0^3 y^2 x\,dy = \int_0^3 y^2 \left(\frac{y}{3}\right)^{\frac{1}{2}} dy = \sqrt{\tfrac{1}{3}} \int_0^3 y^2 \cdot y^{\frac{1}{2}} dy = \sqrt{\tfrac{1}{3}} \int_0^3 y^{\frac{5}{2}} dy$$

$$= \sqrt{\tfrac{1}{3}} \cdot \tfrac{2}{7} \cdot y^{\frac{7}{2}} \Big|_0^3 = 7{,}71$$

$$I_y = \int_A x^2 dA = \int_0^1 x^2 (3-y)\,dx = \int_0^1 x^2 (3 - 3x^2)\,dx = 3\int_0^1 (x^2 - x^4)\,dx$$

$$= 3 \cdot \left[\tfrac{1}{3}x^3 - \tfrac{1}{5}x^5\right]\Big|_0^1 = 3 \cdot \left(\tfrac{1}{3} - \tfrac{1}{5}\right) = 3\left(\tfrac{5}{15} - \tfrac{3}{15}\right) = 3 \cdot \tfrac{2}{15} = \tfrac{2}{5}$$



Powierzchnia elementarna: $dA = x\,dy$

Środek ciężkości powierzchni elementarnej: $\bar{x} = \frac{1}{2}x$, $\bar{y} = y$

Pole powierzchni figury

$A = \int_A dA$ $\qquad y = 2x^2 \qquad x = \left(\frac{1}{2}y\right)^{\frac{1}{2}}$

$$A = \int_0^2 x\,dy = \sqrt{\tfrac{1}{2}} \int_0^2 y^{\frac{1}{2}} dy = \sqrt{\tfrac{1}{2}} \cdot \tfrac{2}{3} \cdot y^{\frac{3}{2}} \Big|_0^2 = 1{,}(3)\,m^2$$

$$\bar{x} = \frac{\int_A \bar{x}\,dA}{\int_A dA} = \frac{\int_0^2 \frac{1}{2}x\,x\,dy}{1{,}(3)} = \frac{\frac{1}{2}\int_0^2 x^2 dy}{1{,}(3)} = \frac{\frac{1}{4}\int_0^2 y\,dy}{1{,}(3)} = \frac{\frac{1}{8}y^2\Big|_0^2}{1{,}(3)} = 0{,}375\,m$$

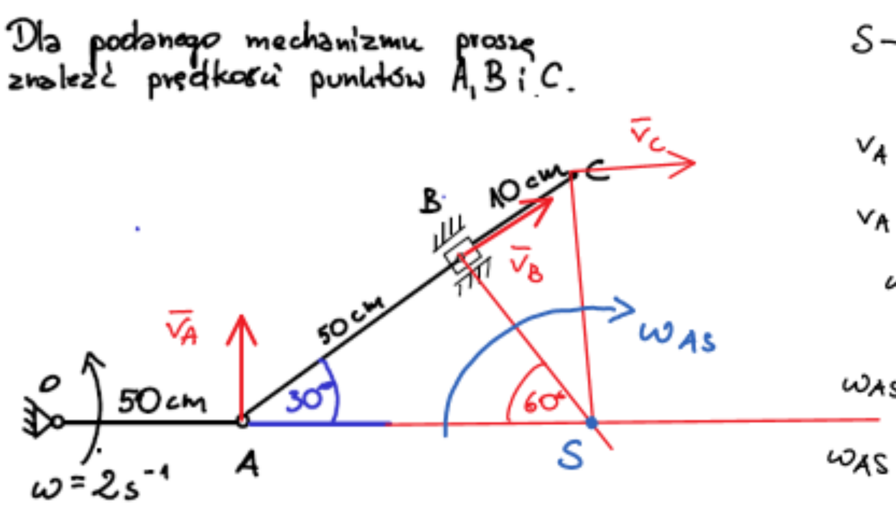
$$\bar{y} = \frac{\int_A \bar{y}\,dA}{\int_A dA} = \frac{\int_0^2 y x\,dy}{1{,}(3)} = \frac{\int_0^2 y \cdot \left(\frac{1}{2}y\right)^{\frac{1}{2}} dy}{1{,}(3)} = \frac{\sqrt{\frac{1}{2}} \int_0^2 y^{\frac{3}{2}} dy}{1{,}(3)} = \frac{\sqrt{\frac{1}{2}} \cdot \frac{2}{5} y^{\frac{5}{2}} \Big|_0^2}{1{,}(3)} = 1{,}2\,m$$

$$I_x = \int_A y^2 dA = \int_0^2 y^2 x\,dy = \sqrt{\tfrac{1}{2}} \int_0^2 y^2 \cdot y^{\frac{1}{2}} dy = \sqrt{\tfrac{1}{2}} \int_0^2 y^{\frac{5}{2}} dy = \sqrt{\tfrac{1}{2}} \cdot \tfrac{2}{7} \cdot y^{\frac{7}{2}} \Big|_0^2 = 2{,}286$$

$$I_y = \int_A x^2 dA = \int_0^1 x^2 (2-y)\,dx = 2\int_0^1 (x^2 - x^4)\,dx = 2\left(\tfrac{1}{3}x^3 - \tfrac{1}{5}x^5\right)\Big|_0^1 = 2 \cdot \left(\tfrac{1}{3} - \tfrac{1}{5}\right) =$$

$$= 2 \cdot \left(\tfrac{5}{15} - \tfrac{3}{15}\right) = 2 \cdot \tfrac{2}{15} = \tfrac{4}{15} = 0{,}27$$

Dla podanego mechanizmu proszę znaleźć prędkości punktów A, B i C.



S – chwilowy środek obrotu

$v_A = \omega \cdot \overline{OA} = 2 \cdot 50 = 100\ cm/s$

$v_A = \omega_{AS} \cdot \overline{AS}$

$\omega_{AS} = \frac{v_A}{AS}$ $\qquad \frac{AB}{AS} = \sin 60°$

$\omega_{AS} = 100 \cdot \frac{3}{100\sqrt{3}}$ $\qquad \frac{AB}{AS} = \frac{\sqrt{3}}{2}$

$\omega_{AS} = \frac{3}{\sqrt{3}} = \sqrt{3}\ s^{-1}$ $\qquad \frac{2AB}{\sqrt{3}} = AS$

$AS = \frac{100}{\sqrt{3}} = \frac{100\sqrt{3}}{3}\ cm$

$v_B = BS \cdot \omega_{AS} = \frac{50\sqrt{3}}{3} \cdot \sqrt{3} = 50\ cm/s$

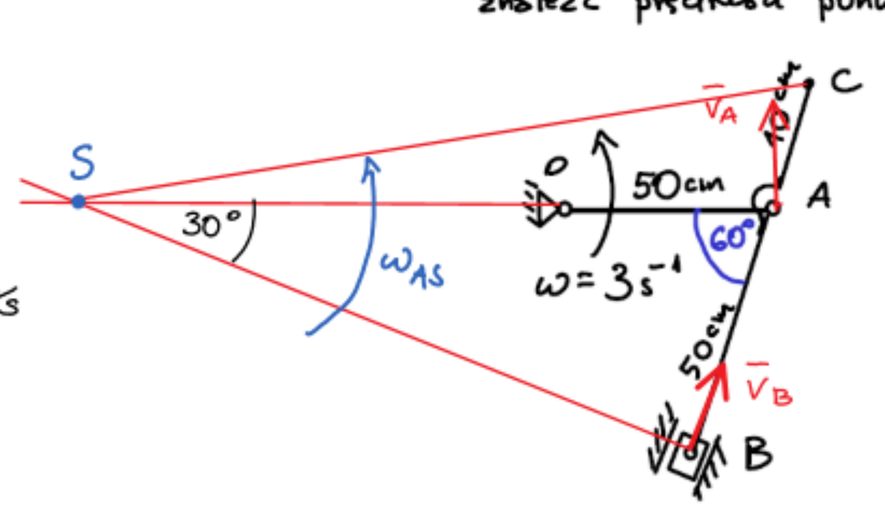
$\frac{BS}{\sin 30} = \frac{AB}{\sin 60} = \frac{AS}{\sin 90}$ $\qquad BS = \frac{AB}{\sqrt{3}} = \frac{50}{\sqrt{3}} = \frac{50\sqrt{3}}{3}$

$2BS = \frac{2AB}{\sqrt{3}} = AS$

$CS = \sqrt{BS^2 + BC^2} = \sqrt{\left(\frac{50\sqrt{3}}{3}\right)^2 + 100} = 30{,}55\ cm$

$v_C = 52{,}91\ cm/s$

Dla podanego mechanizmu proszę znaleźć prędkości punktów A, B i C.



S – chwilowy środek obrotu

$\frac{AB}{AS} = \sin 30°$

$AB = \frac{1}{2}AS$

$AS = 100\,cm$

$v_A = \omega \cdot \overline{OA} = 3 \cdot 50 = 150\ cm/s$

$v_A = \omega_{AS} \cdot AS$

$\omega_{AS} = \frac{v_A}{AS} = \frac{150}{100} = \frac{3}{2}$

$\frac{BS}{AS} = \sin 60°$

$BS = \frac{\sqrt{3}}{2}AS = 50\sqrt{3}\ cm$

$v_B = BS \cdot \omega_{AS} = 50\sqrt{3} \cdot \frac{3}{2} = 75\sqrt{3}\ cm/s = 129{,}9\ cm/s$

$CS = \sqrt{BS^2 + BC^2} = \sqrt{(50\sqrt{3})^2 + 3600} = 105{,}35$

$v_C = CS \cdot \omega_{AS} = 158{,}03\ cm/s$